1. Scope

1.1 This practice covers only $S$-$N$ and $\varepsilon$-$N$ relationships that may be reasonably approximated by a straight line (on appropriate coordinates) for a specific interval of stress or strain. It presents elementary procedures that presently reflect good practice in modeling and analysis. However, because the actual $S$-$N$ or $\varepsilon$-$N$ relationship is approximated by a straight line only within a specific interval of stress or strain, and because the actual fatigue life distribution is unknown, it is not recommended that (a) the $S$-$N$ or $\varepsilon$-$N$ curve be extrapolated outside the interval of testing, or (b) the fatigue life at a specific stress or strain amplitude be estimated below approximately the fifth percentile ($P = 0.05$). As alternative fatigue models and statistical analyses are continually being developed, later revisions of this practice may subsequently present analyses that permit more complete interpretation of $S$-$N$ and $\varepsilon$-$N$ data.

2. Referenced Documents

2.1 ASTM Standards:

E206 Definitions of Terms Relating to Fatigue Testing and the Statistical Analysis of Fatigue Data; Replaced by E1150 (Withdrawn 1988)

E468 Practice for Presentation of Constant Amplitude Fatigue Test Results for Metallic Materials

E513 Definitions of Terms Relating to Constant-Amplitude, Low-Cycle Fatigue Testing; Replaced by E1150 (Withdrawn 1988)

E606/E606M Test Method for Strain-Controlled Fatigue Testing

3. Terminology

3.1 The terms used in this practice shall be used as defined in Definitions E206 and E513. In addition, the following terminology is used:

3.1.1 dependent variable—the fatigue life $N$ (or the logarithm of the fatigue life).

3.1.1.1 Discussion—Log ($N$) is denoted $Y$ in this practice.

3.1.2 independent variable—the selected and controlled variable (namely, stress or strain). It is denoted $X$ in this practice when plotted on appropriate coordinates.

3.1.3 log-normal distribution—the distribution of $N$ when log ($N$) is normally distributed. (Accordingly, it is convenient to analyze log ($N$) using methods based on the normal distribution.)

3.1.4 replicate (repeat) tests—nominally identical tests on different randomly selected test specimens conducted at the same nominal value of the independent variable $X$. Such replicate or repeat tests should be conducted independently; for example, each replicate test should involve a separate set of the test machine and its settings.

3.1.5 run out—no failure at a specified number of load cycles (Practice E468).

3.1.5.1 Discussion—The analyses illustrated in this practice do not apply when the data include either run-outs (or suspended tests). Moreover, the straight-line approximation of the $S$-$N$ or $\varepsilon$-$N$ relationship may not be appropriate at long lives when run-outs are likely.

3.1.5.2 Discussion—For purposes of statistical analysis, a run-out may be viewed as a test specimen that has either been removed from the test or is still running at the time of the data analysis.

4. Significance and Use

4.1 Materials scientists and engineers are making increased use of statistical analyses in interpreting $S$-$N$ and $\varepsilon$-$N$ fatigue data. Statistical analysis applies when the given data can be reasonably assumed to be a random sample of (or representation of) some specific defined population or universe of material of interest (under specific test conditions), and it is desired either to characterize the material or to predict the performance of future random samples of the material (under similar test conditions), or both.
5. Types of S-N and ε-N Curves Considered

5.1 It is well known that the shape of S-N and ε-N curves can depend markedly on the material and test conditions. This practice is restricted to linear or linearized S-N and ε-N relationships, for example,

\[
\log N = A + B (S) \quad \text{or} \quad (1)
\]

\[
\log N = A + B (\varepsilon) \quad \text{or} \quad (2)
\]

\[
\log N = A + B (\log S) \quad \text{or}
\]

in which \( S \) and \( \varepsilon \) may refer to (a) the maximum value of constant-amplitude cyclic stress or strain, given a specific value of the stress or strain ratio, or of the minimum cyclic stress or strain, (b) the amplitude or the range of the constant-amplitude cyclic stress or strain, given a specific value of the mean stress or strain, or (c) analogous information stated in terms of some appropriate independent (controlled) variable.

**NOTE 1**—In certain cases, the amplitude of the stress or strain is not constant during the entire test for a given specimen. In such cases some effective (equivalent) value of \( S \) or \( \varepsilon \) must be established for use in analysis.

5.1.1 The fatigue life \( N \) is the dependent (random) variable in S-N and ε-N tests, whereas \( S \) or \( \varepsilon \) is the independent (controlled) variable.

**NOTE 2**—In certain cases, the independent variable used in analysis is not literally the variable controlled during testing. For example, it is common practice to analyze low-cycle fatigue data treating the range of plastic strain as the controlled variable, when in fact the range of total strain was actually controlled during testing. Although there may be some question regarding the exact nature of the controlled variable in certain S-N and ε-N tests, there is never any doubt that the fatigue life is the dependent variable.

**NOTE 3**—In plotting S-N and ε-N curves, the independent variables \( S \) and \( \varepsilon \) are plotted along the ordinate, with life (the dependent variable) plotted along the abscissa. Refer, for example, to Fig. 1.

5.1.2 The distribution of fatigue life (in any test) is unknown (and indeed may be quite complex in certain situations). For the purposes of simplifying the analysis (while maintaining sound statistical procedures), it is assumed in this practice that the logarithms of the fatigue lives are normally distributed, that is, the fatigue life is log-normally distributed, and that the variance of log life is constant over the entire range of the independent variable used in testing (that is, the scatter in log

---

**FIG. 1** Fitted Relationship Between the Fatigue Life \( N (Y) \) and the Plastic Strain Amplitude \( \Delta \varepsilon_p/2 (X) \) for the Example Data Given

*Note*—The 95% confidence band for the \( \varepsilon-N \) curve as a whole is based on Eq 10. (Note that the dependent variable, fatigue life, is plotted here along the abscissa to conform to engineering convention.)
Eq 3 is used in subsequent analysis. It may be stated more precisely as \( \mu_{Y|x} = A + BX \), where \( \mu_{Y|x} \) is the expected value of \( Y \) given \( X \).

**NOTE 4**—For testing the adequacy of the linear model, see §8.2.

**NOTE 5**—The expected value is the mean of the conceptual population of all \( Y \)’s given a specific level of \( X \). (The median and mean are identical for the symmetrical normal distribution assumed in this practice for \( Y \)).

### 6. Test Planning

6.1 Test planning for \( S-N \) and \( \varepsilon-N \) test programs is discussed in Chapter 3 of Ref (1).\(^4\) Planned grouping (blocking) and randomization are essential features of a well-planned test program. In particular, good test methodology involves use of planned grouping to (a) balance potentially spurious effects of nuisance variables (for example, laboratory humidity) and (b) allow for possible test equipment malfunction during the test program.

**NOTE 6**—A random sampling procedure provides each specimen that conceivably could be selected (tested) an equal (or known) opportunity of actually being selected at each stage of the sampling process. Thus, it is poor practice to use specimens from a single source (plate, heat, supplier) when seeking a random sample of the material being tested unless that particular source is of special interest.

**NOTE 7**—Procedures for using random numbers to obtain random samples and to assign stress or strain amplitudes to specimens (and to establish the time order of testing) are given in Chapter 4 of Ref (2).

#### 7. Sampling

7.1 It is vital that sampling procedures be adopted that assure a random sample of the material being tested. A random sample is required to state that the test specimens are representative of the conceptual universe about which both statistical and engineering inference will be made.

**NOTE 6**—A random sampling procedure provides each specimen that conceivably could be selected (tested) an equal (or known) opportunity of actually being selected at each stage of the sampling process. Thus, it is poor practice to use specimens from a single source (plate, heat, supplier) when seeking a random sample of the material being tested unless that particular source is of special interest.

**NOTE 7**—Procedures for using random numbers to obtain random samples and to assign stress or strain amplitudes to specimens (and to establish the time order of testing) are given in Chapter 4 of Ref (2).

#### 7.1.1 Sample Size—The minimum number of specimens required in \( S-N \) (and \( \varepsilon-N \)) testing depends on the type of test program conducted. The following guidelines given in Chapter 3 of Ref (1) appear reasonable.

---

\(^4\) The boldface numbers in parentheses refer to the list of references appended to this standard.
the symbol “overbar” (\(\bar{\cdot}\)) denotes average (for example, \(\bar{Y} = \sum_{i=1}^{k} Y_i/k\) and \(\bar{X} = \sum_{i=1}^{k} X_i/k\)), \(Y_i = \log N_i, X_i = S_i\) or \(e_i\), or \(\log S_i\) or \(\log e_i\) (refer to Eq 1 and Eq 2), and \(k\) is the total number of test specimens (the total sample size). The recommended expression for estimating the variance of the normal distribution for \(\log N\) is

\[
\sigma^2 = \frac{1}{k-2} \sum_{i=1}^{k} (Y_i - \bar{Y})^2
\]

in which \(\bar{Y} = \bar{A} + \hat{B}X_i\) and the \((k - 2)\) term in the denominator is used instead of \(k\) to make \(\hat{\sigma}^2\) an unbiased estimator of the normal population variance \(\sigma^2\).

Note 8—An assumption of constant variance is usually reasonable for notched and joint specimens up to about 10^6 cycles to failure. The variance of unnotched specimens generally increases with decreasing stress (strain) level (see Section 9). If the assumption of constant variance appears to be dubious, the reader is referred to Ref (3) for the appropriate statistical test.

8.1.1 Confidence Intervals for Parameters \(A\) and \(B\)—The estimators \(\hat{A}\) and \(\hat{B}\) are normally distributed with expected values \(A\) and \(B\), respectively, (regardless of total sample size \(k\)) when conditions (a) through (e) in 8.1 are met. Accordingly, confidence intervals for parameters \(A\) and \(B\) can be established using the \(t\) distribution, Table 1. The confidence interval for \(A\) is given by \(\hat{A} \pm t_p\sigma_A\), or

\[
\hat{A} \pm t_p \sigma_A \left[ \frac{1}{k} + \frac{(\bar{X} - \hat{X})^2}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \right]^{1/2},
\]

and for \(B\) is given by \(\hat{B} \pm t_p\sigma_B\), or

\[
\hat{B} \pm t_p \sigma_B \left[ \frac{1}{k} \frac{(\bar{X} - \hat{X})^2}{\sum_{i=1}^{k} (X_i - \bar{X})^2} \right]^{1/2},
\]

in which the value of \(t_p\) is read from Table 1 for the desired value of \(P\), the confidence level associated with the confidence interval. This table has one entry parameter (the statistical degrees of freedom, \(n\), for \(t\)). For Eq 7 and Eq 8, \(n = k - 2\).

Note 9—The confidence intervals for \(A\) and \(B\) are exact if conditions (a) through (e) in 8.1 are met exactly. However, these intervals are still reasonably accurate when the actual life distribution differs slightly from the (two-parameter) log-normal distribution, that is, when only condition (d) is not met exactly, due to the robustness of the \(t\)-statistic.

Note 10—Because the actual median S-N or e-N relationship is only approximately by a straight line within a specific interval of stress or strain, confidence intervals for \(A\) and \(B\) that pertain to confidence levels greater than approximately 0.95 are not recommended.

8.1.1.1 The meaning of the confidence interval associated with, say, Eq 8 is as follows (Note 11). If the values of \(t_p\) given in Table 1 for, say, \(P = 95\%\) are used in a series of analyses involving the estimation of \(B\) from independent data sets, then in the long run we may expect 95\% of the computed intervals to include the value \(B\). If in each instance we were to assert that \(B\) lies within the interval computed, we should expect to be correct 95 times in 100 and in error 5 times in 100: that is, the statement “\(B\) lies within the computed interval” has a 95\% probability of being correct. But there would be no operational meaning in the following statement made in any one instance: “The probability is 95\% that \(B\) falls within the computed interval in this case” since \(B\) either does or does not fall within the interval. It should also be emphasized that even in independent samples from the same universe, the intervals given by Eq 8 will vary both in width and position from sample to sample. (This variation will be particularly noticeable for small samples.) It is this series of (random) intervals “fluctuating” in size and position that will include, ideally, the value \(B\) 95 times out of 100 for \(P = 95\%\). Similar interpretations hold for confidence intervals associated with other confidence levels. For a given total sample size \(k\), it is evident that the width of the confidence interval for \(B\) will be a minimum whenever

\[
\sum_{i=1}^{k} (X_i - \bar{X})^2
\]

is a maximum. Since the \(X_i\) levels are selected by the investigator, the width of confidence interval for \(B\) may be reduced by appropriate test planning. For example, the width of the interval will be minimized when, for a fixed number of available test specimens, \(k\) half are tested at each of the extreme levels \(X_{\text{min}}\) and \(X_{\text{max}}\). However, this allocation should be used only when there is strong \textit{a priori} knowledge that the S-N or e-N curve is indeed linear—because this allocation precludes a statistical test for linearity (8.2). See Chapter 3 of Ref (1) for a further discussion of efficient selection of stress (or strain) levels and the related specimen allocations to these stress (or strain) levels.

Note 11—This explanation is similar to that of STP 313 (4).

8.1.2 Confidence Band for the Entire Median S-N or e-N Curve (that is, for the Median S-N or e-N Curve as a Whole)—If conditions (a) through (e) in 8.1 are met, an exact confidence band for the entire median S-N or e-N curve (that is, all points on the linear or linearized median S-N or e-N curve considered simultaneously) may be computed using the following equation:

**TABLE 1 Values of \(t_p\) (Abstracted from STP 313 (4))**

<table>
<thead>
<tr>
<th>(r)</th>
<th>(P,%)</th>
<th>(t_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>90</td>
<td>2.1318</td>
</tr>
<tr>
<td>5</td>
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<td>6</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
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<td>18</td>
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</tr>
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<td>21</td>
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</tr>
<tr>
<td>22</td>
<td>1.7171</td>
<td>2.0739</td>
</tr>
</tbody>
</table>

\(a\) \(n\) is not sample size, but the degrees of freedom of \(t\), that is, \(n = k - 2\).

\(b\) \(P\) is the probability in percent that the random variable \(t\) lies in the interval from \(-t_p\) to \(+t_p\).
\[
\hat{A} + \hat{B}X \pm \sqrt{2} F_{p} \beta \left[ \frac{1}{k} + \frac{1}{n} \sum (x - \bar{x})^2 \right]^{1/2}
\]

(10)
in which \( F_{p} \) is given in Table 2. This table involves two entry parameters (the statistical degrees of freedom \( n_{1} \) and \( n_{2} \) for \( F \)). For Eq 9, \( n_{1} = 2 \) and \( n_{2} = (k - 2) \). For example, when \( k = 7 \), \( F_{0.95} = 5.7861 \).

8.1.2.1 A 95 % confidence band computed using Eq 10 is plotted in Fig. 1 for the example data of 8.1.1. Namely, if conditions (a) through (e) are met, and if the values of \( F_{p} \) given in Table 2 for, say, \( P = 95 \% \) are used in a series of analyses involving the construction of confidence bands using Eq 10 for the entire range of \( X \) used in testing; then in the long run we may expect 95 % of the computed hyperbolic bands to include the straight line \( \mu_{Y|x} = A + BX \) everywhere along the entire range of \( X \) used in testing.

NOTE 12—Because the actual median \( S-N \) or \( e-N \) relationship is only approximated by a straight line within a specific interval of stress of strain, confidence bands which pertain to confidence levels greater than approximately 0.95 are not recommended.

8.1.2.2 While the hyperbolic confidence bands generated by Eq 9 and plotted in Fig. 1 are statistically correct, straight-line confidence and tolerance bands parallel to the fitted line \( \mu_{Y|x} = A + BX \) are sometimes used. These bands are described in Chapter 5 of Ref (2).

8.2 Testing the Adequacy of the Linear Model—In 8.1, it was assumed that a linear model is valid, namely that \( \mu_{Y|x} = A + BX \). If the test program is planned such that there is more than one observed value of \( Y \) at some of the \( X_{i} \) levels where \( i \geq 3 \), then a statistical test for linearity can be made based on the \( F \) distribution, Table 2. The log life of the \( j \)th replicate specimen tested in the \( i \)th level of \( X \) is subsequently denoted \( Y_{ij} \).

8.2.1 Suppose that fatigue tests are conducted at \( l \) different levels of \( X \) and that \( m_{i} \) replicate values of \( Y \) are observed at each \( X_{i} \). Then the hypothesis of linearity (that \( \mu_{Y|x} = A + BX \)) is rejected when the computed value of

\[
\frac{\sum_{i} l_{j} (\hat{Y}_{ij} - \hat{\mu}_{ij})^2/(l - 1)}{\sum_{i} l_{j} (Y_{ij} - \hat{\mu}_{ij})^2/(k - l)}
\]

(11)
exceeds \( F_{p} \), where the value of \( F_{p} \) is read from Table 2 for the desired significance level. (The significance level is defined as the probability in percent of incorrectly rejecting the hypothesis of linearity when there is indeed a linear relationship between \( X \) and \( \mu_{Y|x} \).) The total number of specimens tested, \( k \), is computed using

\[
k = \sum_{i} l_{j}
\]

(12)

### Table 2 Values of \( F_{p} \) (Abstracted from STP 313 (4))

<table>
<thead>
<tr>
<th>Degrees of Freedom, ( n_{1} )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tr>
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<td>199.50</td>
<td>215.71</td>
<td>224.58</td>
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<td>19.164</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
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<td>8.6831</td>
<td>6.3589</td>
<td>5.4170</td>
<td>4.893</td>
</tr>
</tbody>
</table>

\(^{A}\) In each row, the top figures are values of \( F \) corresponding to \( P = 95 \% \), the bottom figures correspond to \( P = 99 \% \). Thus, the top figures pertain to the 5 % significance level, whereas the bottom figures pertain to the 1 % significance level. (The bottom figures are not recommended for use in Eq 10.)
Table 2 involves two entry parameters (the statistical degrees of freedom \(n_1\) and \(n_2\) for \(F\)). For Eq 11, \(n_1 = (l - 2)\), and \(n_2 = (k - l)\). For example, \(F_{0.95} = 6.9443\) when \(k = 8\) and \(l = 4\).

8.3.1.2 First, restate (transform) the data in terms of logarithms (base 10 used in this practice due to its wide use in computer software which calculates a value of \(\delta\)).

8.3.1.3 Then, from Eq 4 and Eq 5:

\[ A = -0.24474 \]
\[ B = -1.45144 \]

Also, from Eq 6:

\[ \delta^2 = 0.07837/7 = 0.011195 \] (14)

or,

\[ \delta = 0.1058 \] (15)

8.3.1.4 Accordingly, using Eq 7, the 95 % confidence interval for \(A\) is \((a = -0.2569, 0.1540)\), and, using Eq 8, the 95 % confidence interval for \(B\) is \([-1.6054, -1.2974]\).

8.3.1.5 The fitted line \(\hat{Y} = \log N = -0.24474 - 1.45144 \log (\Delta e_p / 2)\) is displayed in Fig. 1, where the 95 % confidence band computed using Eq 10 is also plotted. (For example, when \(\Delta e_p / 2 = 0.01, \hat{X} = 2.000, \hat{Y}_0 = 2.65814, \hat{Y}_{\text{lower}} = 2.65814 - 0.15215 = 2.50599, \text{and } \hat{Y}_{\text{up}},\) per band = \(2.65814 + 0.15215 = 2.81029\.)

8.3.1.6 The fitted line can be transformed to the form given in Appendix X1 of Practice E606/E606M as follows:

\[ \log N = -0.24474 - 1.45144 \log (\Delta e_p / 2) \]

\[ \log(\Delta e_p / 2) = -0.16862 - 0.68897 \log N \]

\[ \Delta e_p / 2 = 0.67823 (N)^{-0.68897} \]

Substituting cycles (N) to reversals (2N) gives

\[ \Delta e_p / 2 = 0.67823 (1/2)^{-0.68897} (2N)^{-0.68897} \]

\[ \Delta e_p / 2 = 1.09340 (2N)^{-0.68897} \]

The above alternative equation is shown on Fig. 1.

8.3.1.7 Ancillary Calculations:

\[ \hat{X} = -2.53172 \quad \hat{Y} = 3.42990 \] (18)

\[ \sum_{i=1}^{9} (X_i - \bar{X})^2 = 2.63892 \] (19)

\[ \sum_{i=1}^{9} (X_i - \bar{X})(Y_i - \bar{Y}) = -3.8023 \] (20)

\[ \delta_{\hat{X}} = \delta \left( \frac{1}{9} \frac{(-2.53172)^2}{2.63892} \right)^{1/2} = 0.1686 \] (21)

\[ \delta_{\hat{Y}} = \delta \left(2.63892 \right)^{1/2} = 0.06513 \] (22)

8.3.1.8 Test for linearity at the 5 % significance level.

8.3.1.9 We shall ignore the slight differences among the amplitudes of plastic strain and assume that \(l = 4\) and \(k = 9\). Then, at each of the four \(X_i\) levels, we shall compute \(\hat{Y}_i\) using \(\hat{Y}_i = -0.24474 - 1.45144X_i\) and \(\hat{Y}_i\) using \(\hat{Y}_i = \sum Y_i / m_i\). Accordingly, \(F_{0.95} = 5.79\), whereas \(F\) computed (using Eq 11) is 3.62. Hence, we do not reject the linear model in this example.

8.3.1.10 Ancillary Calculations:

**Numerator (\(F\)) = 0.0532/2**

Denominator (\(F\)) = 0.0368/5

8.3.2 Example 2: Consider the following low-cycle fatigue data (also taken from a 1976 E09.08 round-robin test program (laboratory 43)):
8.3.2.1 The $F$ test (Eq 11) in this case indicates that the linear model should be rejected at the 5% significance level (that is, $F$ calculated = 39.36, where $F_{3,5,0.95} = 5.41$). Hence estimation of $A$ and $B$ for the linear model is not recommended. Rather, a nonlinear model should be considered in analysis.

### REFERENCES